Computing Olympic Gold: Ski jumping as an example.

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Abstract – Nowadays, computer science is strongly connected to high-performance sports. Powerful tools for improving performance are simulations and optimizations based on mathematical models as shown here using the example of ski jumping.
Ski jumping has been an Olympic discipline since the first Winter Games in 1924. International competitions are held on three types of hills differing in their hill size, defined as the length between the edge of the take-off ramp and the end of the landing area (L): normal hill ($85 - 109$ m), large hill (≥ 109 m) and ski flying hill (≥ 185 m). The technique has changed several times until now and numerous researchers have analyzed the four interrelated phases: in-run, take-off, flight and landing. At a given hill, the jump length is determined by the initial conditions in-run velocity parallel to the ramp ($\mathbf{v}_0$) and the take-off velocity perpendicular to the ramp ($\mathbf{v}_{\mathrm{p}0}$) as well as the gravitational force $\mathbf{F}_{\mathrm{g}} = m\mathbf{g}$ and the aerodynamic forces drag $\mathbf{F}_{\mathrm{d}} = (\rho/2)D\mathbf{w}^2$ and lift $\mathbf{F}_{\mathrm{l}} = (\rho/2)L\mathbf{w}^2$ that act during the flight on the athlete and his equipment. Besides the air density ρ and the airstream vector $\mathbf{w}$ the aerodynamic forces depend on the drag and lift areas D and L, respectively. They are functions of the equipment, the anthropometrical dimensions and the flight position and can be measured in a wind tunnel. The flight position is characterized by the angle of attack α of the skis relative to the air stream vector, the body-to-ski angle β, the hip angle γ and the *V*-angle between the skis (Fig. 1).

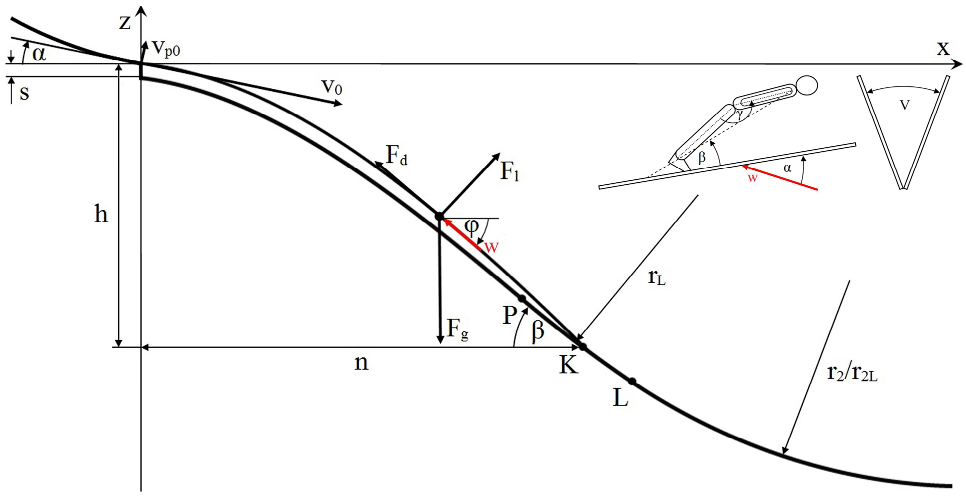

Fig.1: Flight path of a ski jumper on a hill profile. The flight path angle is denoted with φ which is also the angle of the airstream vector $\mathbf{w}$ in calm wind condition. Hill profiles are modeled piecewise and the corresponding hill parameters shown here can be found in the FIS Certificates of Jumping hills.

Since the *V-style* was introduced by Jan Boklöv in 1985, the flight technique has become a predominant performance factor [1]. Throughout the entire flight the athlete has to control his flight position with respect to his individual features so that the pitching moment due to the aerodynamic forces is balanced and ultimately the jump length is maximized. Mathematically speaking this is a constrained optimization problem which the athlete has to solve in fractions of a second.

Taking into account all forces acting during the flight, the motion of a ski jumper can be mathematically modeled by a first order system of coupled differential equations called the equations of motions. Together with a set of initial conditions and the given hill profile this is an initial value problem which has to be solved numerically. Solutions are, among others, the flight trajectory, the jump length and the time courses of both the aerodynamic forces and the flight velocities. Müller et al. (1995) [2] introduced tabulated functions $D = D(t)$ and $L = L(t)$ mapping the time course of the flight position (flight technique) based on field studies and wind tunnel measurements. They can be used to analyze the impact of all performance factors on the jump length [1].
Optimization algorithms can be applied in order to help the athletes to solve the mentioned constrained optimization problem. For this, the drag and lift areas are needed as functions of the flight position. Based on the equations of motion the time course of the flight position that maximizes the jump length within a feasible range is computed.
Remizov [3] was the first to perform optimization studies in ski jumping in 1984. He showed that the angle of attack of the skis α should gradually increase according to a convex function whose form depends on the initial conditions and individual drag and lift areas. However, the flight positions were not constrained and wind tunnel data of a ski jumper who held the skis in parallel to each other were used.
For more realistic optimization studies in *V-style* ski jumping we also included the body-to-ski angle β using a comprehensive set of wind tunnel data [1] and considered flight position constraints. By varying the flight position constraints it has been shown that various time courses of α and β result in similar jump lengths even for one set of drag and lift areas [4]. Denoth et al. (1987) [5] pointed out that the discussion of the optimum flight technique has to include all phases of a ski jump. A small decrease in jump length due to different aerodynamic strategies can easily be masked by the initial velocities $\mathbf{v}_0$ and $\mathbf{v}_{p0}$ which results in a range of flight techniques that have winning potential in a competition. Consequently, the athletes can develop their individual optimum within this range based not only on their individual aerodynamic features but also on individual biomechanical features [4]. This corresponds to a field study during the Winter Olympic Games 2002 which illustrated that the medalists used markedly different flight techniques although two of them had similar anthropometrical dimensions and ski lengths [6]. On the normal and large hills the athletes can use the same flight technique without a significant loss in length but they need to adjust it on the ski flying hill as the in-run velocity parallel to the ramp and thus the aerodynamic forces are considerably larger [4]. However, it has been shown that the take-off velocity perpendicular to the ramp within the range in high-performance ski jumping ($|\mathbf{v}_{p0}| = 2\text{–}3$ ms^{-1}) has a negligibly small impact on the optimum flight technique [7]. Wind changes the airstream vector and thus increases the jump length when blowing up the hill whereas it decreases the jump length when blowing down the hill [1]. On the ski flying hill the effect of wind also needs to be considered for the flight technique optimization [8].
Mathematical models in sports have many advantages, e.g. that ideal experiments can be carried out where it is possible to vary single performance factors without putting the athletes at risk. But the benefit for coaches and athletes depends strongly on the accuracy of the respective models and their inputs.

References

[1] W. Müller, "Determinants of ski-jump performance and implications for health, safety and fairness," *Sports Med.*, vol. 39, no. 2, pp. 85-106, 2009.

[2] W. Müller, D. Platzer and B. Schmölzer, "Scientific approach to ski safety," *Nature*, vol. 375, p. 455, 1995.

[3] L. Remizov, "Biomechanics of optimal flight in ski-jumping," *J Biomech.*, vol. 17, no. 3, pp. 167-171, 1984.

[4] A. Jung, M. Staat and W. Müller, "Flight style optimization in ski jumping on normal, large, and ski flying hills," *J Biomech.*, vol. 47, no. 3, pp. 716-722, 2014.

[5] J. Denoth, S.M. Luethi and H. Gasser, „Methodological problems in optimisation of the flight phase in ski jumping," Int J Sport Biomech, vol. 3, pp. 404-418, 1987.

[6] B. Schmölzer and W. Müller, "Individual flight styles in ski jumping: results obtained during Olympic Games competitions," *J Biomech.*, vol. 38, no. 5, pp. 1055-1065, 2005.

[7] A. Jung, M. Staat and W. Müller,"Optimization of the flight style in ski jumping," *11th World Congress on Computational Mechanics*, July 20-25, Barcelona, Spain, Session on Biomechanics and Applied Dynamics, pp. 799-810, 2014.

[8] A. Jung, M. Staat and W. Müller," Effect of wind on flight style optimisation in ski jumping," *15th International Symposium on Computer Simulation in Biomechanics*, July 9-11, Edinburgh, Scotland, pp. 53-54, 2015.